Hierarchical BOA on Random Decomposable Problems

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Background

Testing optimization algorithms

- Testing on the boundary of the design envelope using artificial test problems.
  - Examples: Concatenated traps, hierarchical traps.
- Testing on classes of random problems.
  - Examples: MAXSAT, Ising spin glass.
- Testing on real-world problems or their approximations.
  - Examples: Military antenna design, cluster optimization.

Objectives

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- Propose random additively decomposable problems (rADPs).
- Three objectives
  - Scalability.
  - Known optimum.
  - Easy generation of random instances.
- Introduce overlap between subproblems to study its effects.
- Test various genetic and evolutionary algorithms on rADPs
  - Hierarchical Bayesian optimization algorithm (hBOA).
  - Genetic algorithm (GA).
  - Univariate marginal distribution algorithm (UMDA).
  - Hill climbing (HC).

Additively Decomposable Problems (rADPs)

Additively Decomposable Problems

Fitness defined as

$$f(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{m} f_i(S_i),$$

where

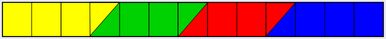
- n is the number of bits (variables),
- m is the number of subproblems,
- S_i is the subset of variables in ith subproblem.

ADPs of Bounded Order With and Without Overlap

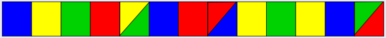
Order-k ADPs with and without overlap

- Each subproblem contains k bits.
- Separable problems contain non-overlapping subproblems:

- There may be overlap in o bits between neighboring subproblems with subproblems defined in contiguous blocks:



- Bits may be shuffled to not enforce tight subproblems.



Verifying optimum of order-k ADPs

- Need to use problem-specific knowledge.
- Use dynamic programming to solve in $O(2^k n)$ evaluations.

Random ADPs (rADPs)

Generating random ADPs

- User sets m, k, and o.
- Randomly generate 2^k values for each subproblem from [0,1).
- Reorder bits randomly.

Test Problems

- Order of subproblems, $k = 4$.
- Overlap $o = 0$, $o = 1$, and $o = 2$.
- Vary problem size to study scalability.
- Generate 1000 random instances for each m, k, and o.

Experiments

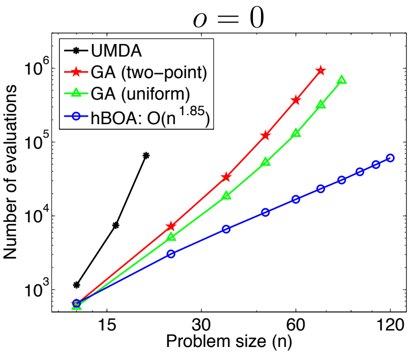
Population-based methods

- Population size
  - Minimum population size to ensure convergence in 10 out of 10 independent runs (by bisection method).
- Selection and replacement strategies
  - Binary tournament selection.
  - Restricted tournament replacement.
- Genetic algorithm
  - One-point or uniform crossover ($p_c = 0.6$), bit-flip mutation ($p_m = 1/n$).

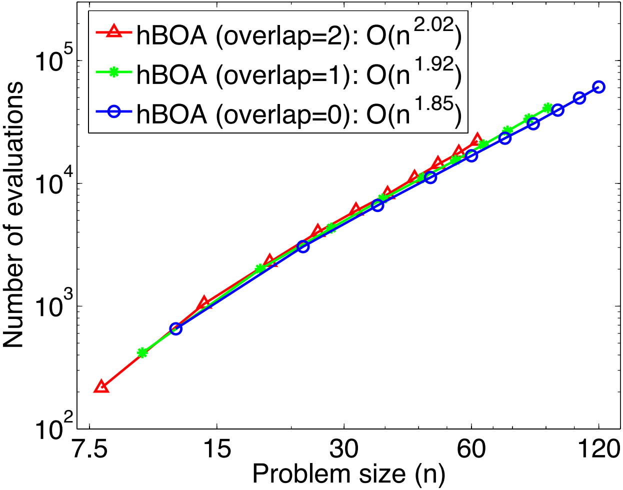
Hill climbing

- Bit-flip mutation with $p_m = k/n$.
- Results averaged over 10 runs.

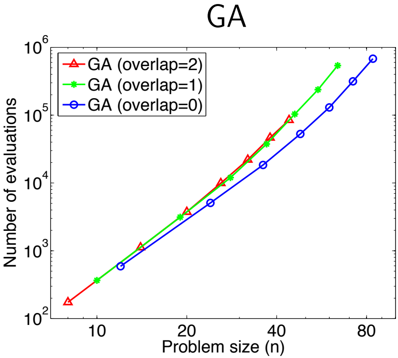
Comparison of Population-Based Methods for $o = 0$ and 2

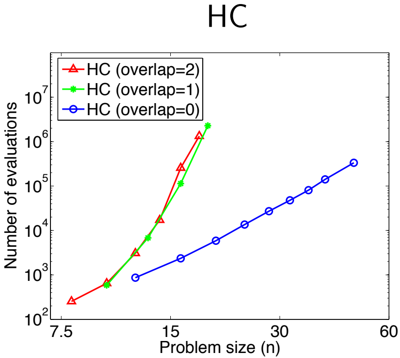


Effects of Overlap on hBOA

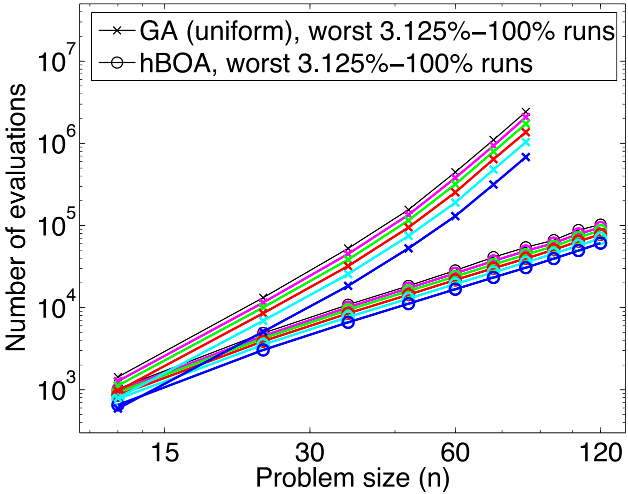


Effects of Overlap on GA and HC





Sensitivity of hBOA and GA to Problem Difficulty



Summary and Conclusions

Summary

- Proposed random additively decomposable problems (rADPs)
  - Problem size, difficulty, and overlap easily controlled.
  - Problem instances deterministically solvable in linear time.
  - Large numbers of random instances can be generated easily.
- Tested hBOA, GA, UMDA, and HC on rADPs.
- Code & papers available at `http://medal.cs.umsl.edu`

Conclusions

- Best performance achieved with hBOA ($O(n^{2.02})$ evaluations).
- Deception not enforced, but linkage learning still important.
- Recombination is less sensitive to overlap than mutation.
- Difficulty of rADPs does not seem to vary much.
- Can be used to thoroughly test other optimization methods.